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RADIATIVE TRANSFER EFFECTS IN NATURAL CONVECTION ABOVE FIRES—TRANSPARENT APPROXIMATION, AMBIENT ATMOSPHERE NON-ISOTHERMAL

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Abstract—This paper describes the effect of radiative heat transfer on a buoyant axisymmetric turbulent plume in an atmosphere with various lapse rates. The fluid is supposed to be optically transparent. The analysis is carried out for a possible range of the values of the parameters expected in such problems and solution curves presented for one case. For purposes of comparison, the values of the heights attained by the plume when the buoyancy is decreased or increased by 50 per cent according as the atmosphere is stable or unstable, are given.

NOMENCLATURE

x, r,	co-ordinates along the axial and radial directions;
u, v,	axial and radial velocity components;
ρ, ρ_∞, T, T_∞, γ, γ_∞,	ordinary densities, temperature and specific weights inside and outside the plume respectively;
$\rho_0, \rho_{\infty 0}$, $T_0, T_{\infty 0}$,	potential densities and temperatures inside and outside the plume;
p, p_0,	pressure in the plume and standard atmospheric pressure;
g,	acceleration due to gravity;
h,	enthalpy per unit mass;
k,	exponent in adiabatic law;
C_p,	average specific heat of the air;
τ, q,	radial turbulent shear and heat stress;
H,	rate of radiative heating per unit volume;
k^*,	absorption coefficient per unit volume;
σ,	Stefan's constant;
$\frac{\Delta\gamma_0}{\gamma_0}$,	$\frac{\rho_{\infty 0} - \rho_0}{\rho_0}$.

1. INTRODUCTION

In a recent publication Murgai and Emmons [1] solved the problem of turbulent natural convection above fires in a dry calm atmosphere with an arbitrary lapse rate variation. The results were presented in the form of solution curves. The initial fire size, momentum release rate and atmospheric lapse rate were regarded as independent parameters. In a later publication Murgai [2] examined the influence of radiative heat transfer on this problem. In doing so both the asymptotic forms of the radiative transfer equation, viz. the opaque and the transparent approximations were examined. The solution curves were, however, given for the transparent case and for an atmosphere of constant potential temperature. The present is an attempt to extend the results of this investigation to an atmosphere of arbitrary lapse rate variation. The framework of analysis as also the underlying assumptions are essentially those as contained in [1] and [2] viz. the use of the hydrostatic approximation with regard to the pressure and the well-known Boussinesq approximation according to which the density is assumed to be constant everywhere except in the buoyancy term. In converting density to temperature use has been made of the linear law of variation applicable to incompressible fluids for small temperature changes. Also the energy due to dissipation is neglected. The composition of the plume is assumed to be uniform throughout and no change is envisaged due to variation in temperature or mixing. In other words, it is assumed that the products of the combustion in the plume consist of a single component whose radiation properties—emission and absorption—remain constant with height. Further, the ambient atmosphere is assumed to be calm and emitting black-body radiation. The point which needs reconsideration, in the present context, is the energy conservation relation which is discussed in the section that follows. With such an increase in the scope of this investigation the number of independent parameters become large and to provide solution curves in their entirety becomes rather laborious. We have presented results here for a range of initial temperatures, absorption coefficient (in so far as it remains within the transparent approximation), lapse rate, but for a constant value of the parameter characterizing the initial momentum rate hoping that in problems primarily meant to estimate the radiative transfer effects in the dynamics of the plume this will not alter the results significantly.

The solutions, obtained here, are in conformity with the conclusions arrived at in [1] and [2]. The results of the former indicate that the plume dies out or moves with increasing velocity according as the outside atmosphere has a stable or unstable lapse rate. The results of the latter, relating to radiative transfer effects on the plume in neutral atmosphere, show that the height for a certain decrease in buoyancy is much less than the corresponding case without radiation transfer. The results of the present investigation indicate that when these two effects viz. lapse rate variation and radiative transfer are considered simultaneously they act in unison when the lapse rate is stable in the ambient air thus making the plume die out at a still lesser height than it would when either one is present. When the lapse rate is unstable these two effects act in opposition. These facts are explained, in greater detail, towards the end of the paper.

2. FUNDAMENTAL EQUATIONS

The conservation equations of mass, momentum and energy for an axially-symmetric turbulent gravitational convection column with radiative transfer are respectively

$$\frac{\partial}{\partial x}(\gamma r u) + \frac{\partial}{\partial r}(\gamma r v) = 0, \qquad (2.1)$$

$$r\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = r\left(\frac{\gamma_\infty - \gamma}{\rho}\right) + \frac{1}{\rho}\frac{\partial}{\partial r}(r\tau), \qquad (2.2)$$

$$r\rho\left(u\frac{\partial h}{\partial x} + v\frac{\partial h}{\partial r} + ug\right) = \frac{\partial}{\partial r}(rq) + rH. \qquad (2.3)$$

Here u and v are the averaged velocity components along the axes x and r in the axial and radial directions. The density ρ and all other fluid properties are assumed to be local mean values. $\gamma = \rho g$ is the specific weight of the medium, and g the acceleration due to gravity. γ_∞ is the value of γ far removed from the plume. $h = C_p T$ is the enthalpy per unit mass C_p being the specific heat at constant pressure. The turbulent shear stress and the heat flux are $\tau = -\rho\overline{u'v'}$ and $q = -\rho C_p\overline{v'T'}$ respectively where u', v', T' are the fluctuating parts of the velocity components and temperature T (in absolute scale). H is the heating rate per unit volume due to radiation and is given by the relation

$$H = -4\pi k^* B + k^* \int I \, d\omega, \qquad (2.4)$$

where I is the integrated intensity of radiation, K^* the grey absorption coefficient for the fluid, B the Planck function and $d\omega$ an element of solid angle. The quantity H in equation as (2.3), as mentioned above, stands for the radiative heating rate per unit volume at any point in the plume and consists of the net balance of the radiative energy emitted by it and that received from the rest of the surroundings. In the equation (2.4) the first term on the right-hand side gives the emission part and the second gives the part received. For a transparent medium, to which this present investigation applies, $1/k^*$, the mean free path is very much larger than a characteristic length say the plume size. For the present case the contribution to this integral may be assumed to be entirely due to the atmosphere exterior to the plume. With this approximation as the basis the integral may be readily evaluated. The problem, studied here, has an axial symmetry and if we assume isotropy with respect to the radiation field we have

$$\int I \, dw = 4\pi I = 4\sigma T_\infty^4, \qquad (2.5)$$

where T_∞ is the temperature outside the plume and σ the Stefan–Boltzmann constant. From the relations (2.4) and (2.5) we get

$$H = -4\sigma k^*(T^4 - T^4_\infty). \tag{2.6}$$

H in equation (2.3) is now replaced by equation (2.6). The ordinary temperature and density are expressed in terms of the corresponding potential quantities as given in (1). The conservation equations thus obtained, are integrated with respect to r between the limits 0 to ∞. Thus we get

$$\frac{d}{dx}(b_m^2 u_m) = a\, b_m u_m, \tag{2.7}$$

$$\frac{d}{dx}(b_m^2 u_m^2) = g\, b_m^2 \theta_m, \tag{2.8}$$

$$\frac{d}{dx}(b_m^2 u_m \theta_m) + b_m^2 u_m \frac{d}{dx}(\ln T_{\infty 0}) = -\frac{4\sigma k^*(p/p_0)^{3k-4/k}}{\rho_0 C_p} T^3_{\infty 0} b_m^2 [(1+\theta_m)^4 - 1] \tag{2.9}$$

where

$$u_m = \frac{\int_0^\infty ru^2\, dr}{\int_0^\infty ru\, dr},\ b_m = \frac{2}{u_m}\int_0^\infty ru\, dr,$$

and

$$\theta_m = \frac{2}{b_m^2}\int_0^\infty r\left(\frac{\Delta\gamma_0}{\gamma_0}\right) dr.$$

The quantities u_m, b_m and θ_m may be considered as defining the equivalent top hat profiles as explained in [1]. The boundary conditions are

$$b_m = b_0,\ u_m = u_{m0},\ \theta_m = \delta, \tag{2.10}$$

at $x = 0$.

The temperature variation in the ambient air for a constant lapse rate Γ is given by the relation

$$T_{\infty 0} = (T_{\infty 0})_0 \exp\left(\frac{\Gamma a \delta}{b_0} x\right), \tag{2.11}$$

where $(T_{\infty 0})_0$ is the value of $T_{\infty 0}$ at $x = 0$.

Now we define the following set of non-dimensional variables:

$$\left.\begin{aligned}
x' &= \frac{ax}{b_0},\ u' = \frac{u_m}{(gb_0\delta/a)^{1/2}},\ b' = \frac{b_m}{b_0},\\
\lambda &= \frac{\theta_m}{\delta},\ \Gamma = \frac{b_0}{a\delta}\frac{d}{dx}(\ln T_{\infty 0}),\\
u_0 &= \frac{u_{m0}}{(gb_0\delta/a)^{1/2}},\\
\delta &= \left(\frac{T - T_\infty}{T_\infty}\right)_{x=0} = \left(\frac{T_0 - T_{\infty 0}}{T_{\infty 0}}\right)_{x=0},\\
\phi &= \frac{4k^*\sigma(T_{\infty 0})_0^3\, b_0^{1/2}\,(p/p_0)^{3k-4/k}}{P_0 C_p \delta^{3/2}(ag)^{1/2}}
\end{aligned}\right\} \tag{2.12}$$

In terms of these non-dimensional variables equations (2.7) to (2.10) after removing the primes reduce to

$$\frac{d}{dx}(b^2 u) = bu, \tag{2.13}$$

$$\frac{d}{dx}(b^2 u^2) = b^2 \lambda, \tag{2.14}$$

$$\frac{d}{dx}(b^2 u\lambda) + \Gamma b^2 u = -\phi b^2[(1+\delta\lambda)^4 - 1] \exp[3\Gamma\delta x] \tag{2.15}$$

$$b = \lambda = 1,\ u = u_0 \tag{2.16}$$

at $x = 0$.

The parameter u_0 except for the entrainment constant a, is the Froude number; the quantity ϕ is characteristic of the radiative heat transfer. The case $\phi = 0$ is discussed in reference [1] while the case $\Gamma = 0$ is the subject matter of [2].

These equations are solved on IBM 1620 computer for various combinations of the parameters Γ, δ, and ϕ, for a constant value of u_0. The results of a few typical cases are shown graphically. To illustrate the effect of radiative heat transfer on the buoyant plume two tables corresponding to different lapse rates, fire sizes, temperatures and varying degree of transparency are constructed. For the positive values of Γ the heights attained by the plume when the buoyancy is reduced to half are given whereas for negative Γ these heights correspond to 50 per cent increase in the buoyancy. The information regarding the rest of the calculations may be supplied on request.

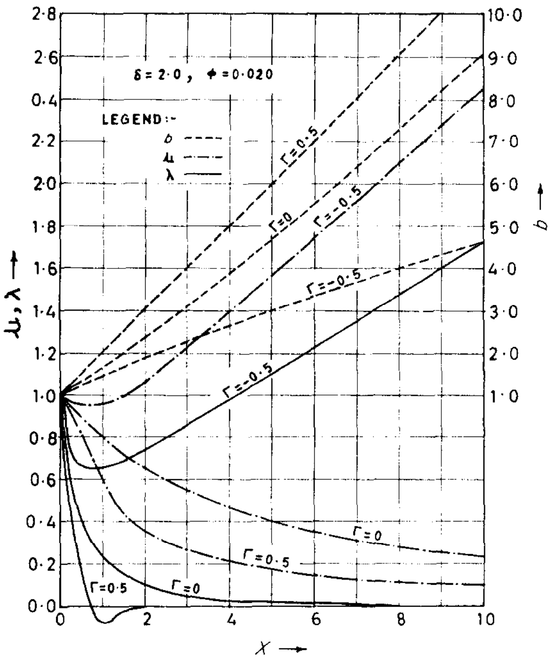

FIG. 1. The variation of plume width b, the vertical velocity u and the buoyancy λ with height x and the lapse rate Γ for $\delta = 2$ and $\phi = 0{\cdot}02$.

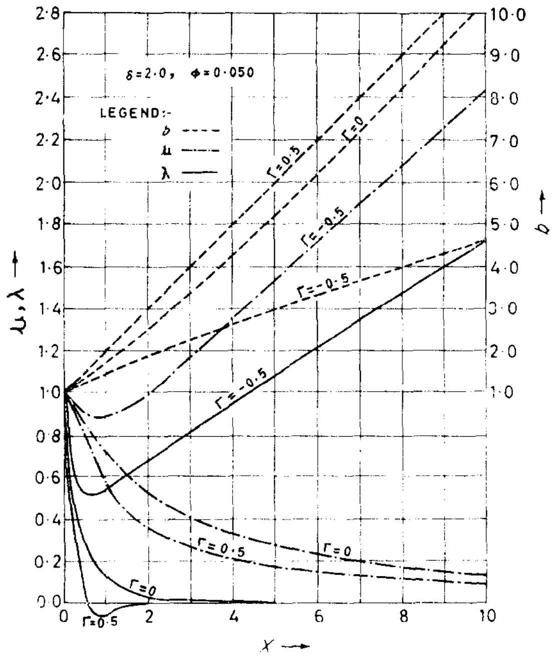

FIG. 2. The variation of plume width b, the vertical velocity u and the buoyancy λ with height x and the lapse rate Γ for $\delta = 2$ and $\phi = 0{\cdot}05$.

3. DISCUSSION

With a view to physically interpret the results presented here it seems worthwhile to go into the "mechanics" of the hot plume in a calm atmosphere with finite lapse rate. In the problem if one does not take the radiative losses into account the only mechanism provided for cooling, in the above formulation, is the process of mixing of the hot plume with the surroundings. When the outside atmosphere has a positive lapse rate the temperature steadily increases upwards. The hot plume, therefore, during its ascent, continuously gets mixed with increasingly hot air from outside with the result that the temperature of the plume and that of the ambient air gets equalized very soon with the rapid loss of buoyancy. If to such a system the radiative loss is added, as an additional mode of cooling of the plume, it is apparent that the buoyancy loss will occur sooner. A comparison of the fifty per cent heights for the corresponding values of ϕ for constant potential temperature lapse rates $\Gamma = 0$ and $\Gamma \neq 0$ respectively given in Table 1 corroborate this conclusion. This process of continuous loss of buoyancy may continue for larger values of Γ, till the latter becomes negative. The plume may, however, still rise due to the residual momentum, the negative buoyancy now opposing this motion. This phase will continue till the velocity or momentum becomes zero. Negative buoyancy means outside temperature is larger than that of the plume. When the mode of radiative transfer is incorporated in the formulation of the problem, as in the present case, the plume starts gaining heat from the surroundings (rather than loosing) by radiation, thus making the buoyancy less and less negative till it becomes zero once again. This recovery phase is indicated by small pips below the x-axis in the λ–x curve for $\Gamma = 0{\cdot}5$ in Figs. 1 and 2. Things are the other way round for a negative temperature lapse rate (Γ negative). In this case the outside temperature decreases with height and the plume in its ascent mixes with a continuously colder air. For moderately low values of Γ, the temperature difference may become

Table 1. *Fifty per cent height h in cm for buoyancy for typical fire sizes b_0 in cm and their initial temperatures T^* in °C for $k^* = 0{\cdot}0$ 1 cm corresponding to some values of the dimensionless parameters ϕ, Γ and δ*

δ	T^*	$b_o \times 10^{-5}$	ϕ	Γ	x	$h \times 10^{-5}$
1·00	327	0·0241	0·02	0·00	0·6429	0·1549
				0·50	0·3602	0·0868
				3·00	0·1249	0·0301
				5·00	0·0829	0·0200
		0·1509	0·05	0·00	0·4832	0·7291
				0·50	0·2975	0·4489
				3·00	0·1136	0·1714
				5·00	0·0774	0·1168
2·00	627	0·1932	0·02	0·00	0·3675	0·7100
				0·50	0·2288	0·4420
				3·00	0·0924	0·1785
				5·00	0·0649	0·1254
		0·1207	0·05	0·00	0·2056	2·482
				0·50	0·1472	1·777
				3·00	0·0703	0·8485
				5·00	0·0516	0·6228
3·00	927	0·6520	0·02	0·00	0·1919	1·251
				0·50	0·1291	0·8417
				3·00	0·0510	0·3325
				5·00	0·0437	0·2849
		4·075	0·05	0·00	0·0900	3·667
				0·50	0·0848	3·457
				3·00	0·0405	1·650
				5·00	0·0312	1·271
4·00	1227	1·545	0·02	0·00	0·1000	1·545
				0·50	0·0847	1·301
				3·00	0·0387	0·5979
				5·00	0·0292	0·4511
		9·659	0·05	0·00	0·050	4·829
				0·50	0·040	3·864
				3·00	0·0242	2·337
				5·00	0·0194	1·874
5·00	1527	3·018	0·02	0·50	0·0750	2·2640
				3·00	0·0258	0·7786
				5·00	0·0201	0·6066
		18·87	0·05	0·00	0·0400	7·548
				0·50	0·0240	4·529
				3·00	0·0150	2·831
				5·00	0·0124	2·340

Table 2. Fifty per cent increase in the height h in cm for buoyancy for typical fire sizes b_o in cm and their initial temperature T^* °C for different values of ϕ, Γ, δ and for $u_{mo} = 10^3$ cm/sec (velocity at $x = 0$)

δ	T^*	$b_o \times 10^{-5}$	ϕ	Γ	x	$h \times 10^{-5}$
				−0·5	6·282	0·0641
			0·00	−1·0	2·694	0·0275
				−1·5	1·012	0·0103
				−0·5	8·065	0·0823
1·0	327	102·0	0·2	−1·0	2·673	0·0273
				−1·5	1·154	0·0118
				−0·5	8·201	0·0837
			0·05	−1·0	2·763	0·0282
				−1·5	1·249	0·0127
				−0·5	6·282	0·0321
			0·00	−1·0	2·694	0·0137
				−1·5	1·012	0·0052
				−0·5	8·123	0·0414
2·0	627	51·02	0·02	−1·0	2·728	0·0139
				−1·5	1·219	0·0062
				−0·5	8·226	0·0420
			0·05	−1·0	2·835	0·0145
				−1·5	1·339	0·0068
				−0·5	6·282	0·0214
			0·00	−1·0	2·694	0·0092
				−1·5	1·012	0·0034
				−0·5	8·185	0·0278
3·0	927	34·01	0·02	−1·0	2·789	0·0095
				−1·5	1·290	0·0044
				−0·5	8·287	0·0282
			0·05	−1·0	2·896	0·0098
				−1·5	1·413	0·0048
				−0·5	6·282	0·0160
			0·00	−1·0	2·694	0·0069
				−1·5	1·012	0·0026
				−0·5	8·234	0·0210
4·0	1227	25·51	0·02	−1·0	2·842	0·0072
				−1·5	1·352	0·0034
				−0·5	8·304	0·0212
			0·05	−1·0	2·938	0·0075
				−1·5	1·464	0·0037
				−0·5	6·282	0·0128
			0·00	−1·0	2·694	0·0055
				−1·5	1·012	0·0021
				−0·5	8·271	0·0169
5·0	1527	20·41	0·02	−1·0	2·882	0·0059
				−1·5	1·401	0·0029
				−0·5	8·351	0·0170
			0·05	−1·0	2·966	0·0061
				−1·5	1·499	0·0031

small hence the loss in buoyancy, resulting in the plume losing its existence at some suitable height. For larger negative values of Γ, however, the inside and outside temperature may never equalize and the plume will never stop! When the plume is radiating its temperature decreases due to this additional mode and tends to equalize with the outside atmosphere. Thus in this case Γ and ϕ, the parameters characterizing the lapse rate and radiation respectively, work in opposite direction and depending on their relative magnitudes the effect of one or the other will predominate. Within the framework of transparent approximation ϕ can take values less than unity. For this case, therefore, even for the largest value of ϕ the qualitative behaviour of the plume will remain the same as for $\phi = 0$ except that these curves for u and λ ($\phi \neq 0$) will lie below the corresponding curves for the same value of Γ. For purposes of comparison we can define a characteristic height for unstable atmosphere which is such that buoyancy increases by fifty per cent from its value at the level $x = 0$. Table 2 gives a comparison of these heights for some typical values of the parameters for radiative and non-radiative cases respectively.

From the nature of the solution it appears that for the opaque case the radiative losses will become increasingly dominant with the opacity of the medium making the effect of the outside atmosphere less important comparatively speaking.

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Résumé—Cet article décrit l'effet du transport de chaleur par rayonnement sur une colonne de gaz produite par convection naturelle turbulente dans une atmosphère avec différentes vitesses de retombée. On suppose que le fluide est transparent. L'analyse est conduite pour une gamme possible de valeurs des paramètres supposés dans de tels problèmes et les solutions sont présentées sous forme de courbes dans un cas. Dans un but de comparaison, on donne les valeurs des hauteurs atteintes par la colonne lorsque la convection naturelle est diminuée ou augmentée de 50 pour cent selon que l'atmosphère est stable ou instable.

Zusammenfassung—Diese Arbeit beschreibt den Einfluss übertragener Strahlungswärme auf eine achsensymmetrische Feder, die sich turbulent in einer Atmosphäre mit verschiedenen Temperaturgefällen über der Höhe wegen wirkender Auftriebskräfte bewegt. Das Medium soll optisch durchsichtig sein. Die Analyse wird für einen bei solchen Problemen zu erwartenden Bereich der Parameter durchgeführt. Für einen Fall werden Lösungskurven aufgezeichnet. Zum Vergleich mit anderen Messungen werden die Höhen angegeben, die von der Feder durch einen um 50 Prozent niedrigeren oder höheren Auftrieb entsprechend einer stabilen oder instabilen Atmosphäre erreicht werden.

Аннотация—В статье рассматривается влияние лучистого теплообмена на восходящую под действием естественной конвекции турбулентную струю в атмосфере при различных вертикальных температурных градиентах. Газ предполагается оптически прозрачным. Анализируестя диапазон возможных значений параметров в такого рода задачах. Аналитические решения представлены для одного случая в виде кривых. Для сравнения приводятся значения высоты, достигаемой струей при уменьшении или увеличении подъемной силы на 50 процеита в зависимости от того, спокойна или неспокойна атмосфера.